

ELASTIC-PLASTIC BENDING OF A RECTANGULAR BEAM

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Annotations. The article under review examines the stress-strain state of a rectangular beam, referred to a support with hinges at both ends, in flat bending under elastoplastic deformations. As a result, a differential equation for the bent axis of the beam was obtained and solved.

Keywords: elastic-plastic deformation, elastic bending, yield strength

With increasing load, the bending moment in the cross section of the beam reaches the value $M_T = \sigma_T W_x$ (σ_T – is the yield strength of the material, W_x – is the axial section modulus), at the most distant points of the section from the neutral axis, the material passes into a plastic state, in accordance with the Prandtl diagram (Fig. 1). Strain begins to increase at constant stress.

With a further increase in the load, plastic deformation gradually occupies an increasing part of the cross section. The diagram of normal stresses divides the section into three zones: in the central elastic core, the stresses change linearly along the ordinate axis, and in the two extreme plastic zones remain constant and equal to $\pm\sigma_T$.

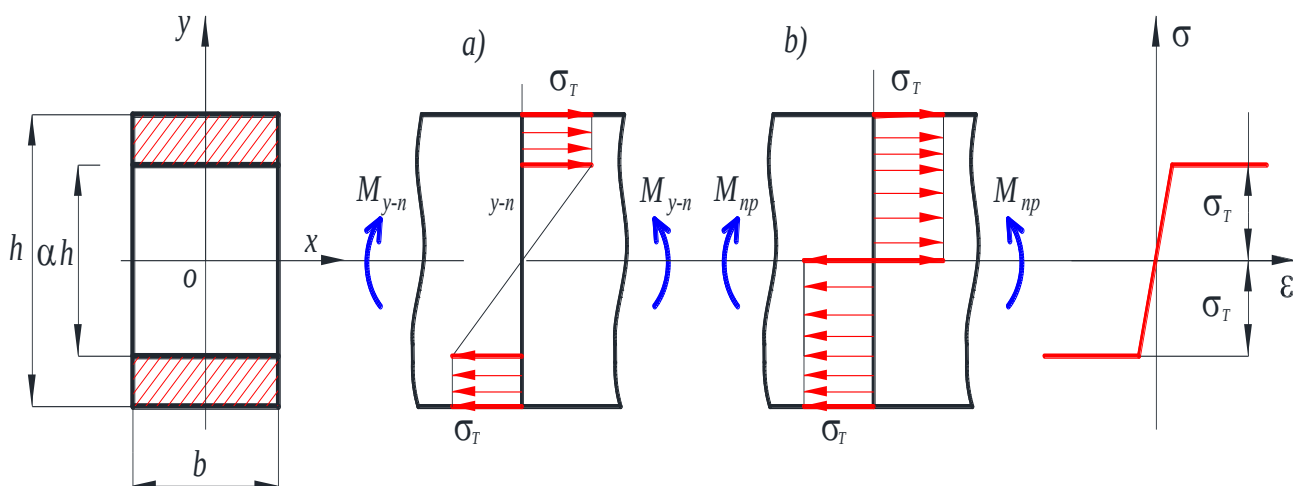


Figure 1. Prandtl diagram [4]

Denote the thickness of the elastic core by αh , depending on the value of the coefficient α , we will distinguish three types of bending:

a) $\alpha = 1$ – elastic bend:

$$M_{y-n} = \sigma W_x, \quad M_{y-n} = M_T = \sigma_T W_x, \quad W_x = \frac{bh^2}{6}$$

b) $\alpha = 0$ – plastic bending: $M_{np} = \sigma_T W_{pl}, \quad W_{pl} = \frac{bh^2}{4}$

this state in which the stresses at all points of the section are equal to the yield strength, is called a plastic hinge, unlike a conventional hinge, it is a constant value equal to the limiting moment M_{np} which corresponds to the complete exhaustion of the bearing capacity of a given section.

c) $0 < \alpha < 1$ – elastoplastic

Let us determine the bending moment during elastic-plastic bending, for which we will use the stress diagram from Fig. 1,

$$M_{y-n} = 2 \left[b \sigma_T (1 - \alpha) \frac{h}{2} \cdot \frac{1}{2} (1 + \alpha) \frac{h}{2} + \frac{1}{2} b \sigma_T \alpha \frac{h}{2} \cdot \frac{2}{3} \alpha \frac{h}{2} \right] = M_{np} \left(1 - \frac{\alpha^2}{3} \right) \quad (1)$$

Find the bent axis of the beam and consider an example. Let two identical, oppositely directed moments be applied to the end sections of a hinged beam (Fig. 2.). In the cross section, normal stresses are distributed according to the following law:

$$\sigma = \begin{cases} E\varepsilon & |y| \leq \frac{\alpha h}{2} \\ \sigma_T & |y| \geq \frac{\alpha h}{2} \end{cases}$$

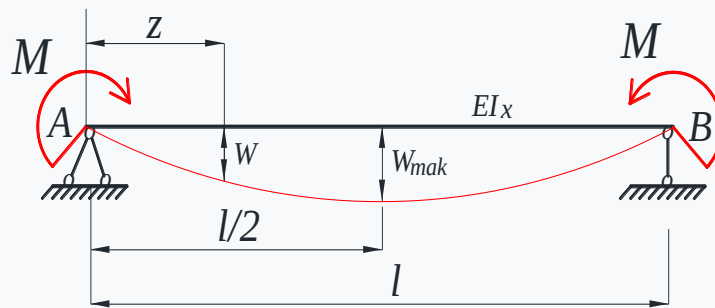


Figure 2. Oppositely directed moments [4]

According to the hypothesis of flat sections, the deformation at the point of the section with the ordinate y ,

$\varepsilon = \frac{y}{\rho}$ — here ρ is the radius of curvature of the curved axis of the beam.

Bending moment in section:

$$M = \int_A \sigma y dA = 2 \int_{\frac{\alpha h}{2}}^{\frac{h}{2}} \sigma_T y dy + \int_{-\frac{\alpha h}{2}}^{\frac{\alpha h}{2}} E \frac{y}{\rho} y dA = 2 \sigma_T \int_{\frac{\alpha h}{2}}^{\frac{h}{2}} y dy + \frac{E}{\rho} \int_{-\frac{\alpha h}{2}}^{\frac{\alpha h}{2}} y^2 dA = 2 \sigma_T S_x^{n,l} + \frac{E}{\rho} I_x^{ynp} \quad (2)$$

Included in this expression $S_x^{n,l}$ — is the static moment of the upper (or lower) plastic zone, and I_x^{ynp} — control is the moment of inertia of the elastic core relative to the neutral axis

$$S_x^{n,l} = \int_{\frac{\alpha h}{2}}^{\frac{h}{2}} b y dy = \frac{b y^2}{2} \Big|_{\frac{\alpha h}{2}}^{\frac{h}{2}} = \frac{b h^2}{8} (1 - \alpha^2); \quad I_x^{ynp} = \frac{b (\alpha h)^3}{12} = \alpha^3 I_x \quad (3)$$

From equation (2) we find the curvature of the curved axis:

$$\frac{1}{\rho} = \frac{M}{EI_x^{ynp}} = \frac{2 S_x^{n,l} \sigma_T}{EI_x^{ynp}} = \frac{M_{np} \left(1 - \frac{\alpha^2}{3} - 1 + \alpha^2 \right)}{\alpha^3 EI_x} = \frac{2 M_{np}}{3 \alpha EI_x} = \frac{2 \sigma_T}{\alpha E h}$$

This formula can be used to determine the stress in an elastic core:

$$\sigma = \frac{E}{\rho} \cdot y = \frac{2 \sigma_T}{\alpha h} \cdot y$$

and in the plastic zone it is constant: $\sigma = \sigma_T$.

Assuming that the displacements are small even under plastic deformations, we write the approximate differential equation of the curved axis:

$$\frac{1}{\rho} = \frac{d^2 W}{dz^2} = \frac{2\sigma_T}{\alpha E h} \quad (4)$$

Separating the variables of the differential equation and integrating twice, we obtain the following equations for the angle of rotation v and deflection W :

$$v = \frac{dW}{dz} = \frac{2\sigma_T}{\alpha E h} \cdot z + C_1$$

$$W = \frac{\sigma_T}{\alpha E h} \cdot z^2 + C_1 \cdot z + C_2$$

The constants of integration are found from the boundary conditions: For $z = 0$ and $z = l \rightarrow W = 0$

Substituting the found constants $C_1 = -\frac{\sigma_T}{\alpha E h} \cdot l$ and $C_2 = 0$, we finally obtain the equations for the angle of rotation and deflection:

$$v = \frac{\sigma_T}{\alpha E h} (2z - l); \quad W = \frac{\sigma_T}{\alpha E h} (z^2 - lz)$$

Maximum deflection (in the middle of the beam, $z = \frac{l}{2}$):

$$W = -\frac{1}{4} \frac{\sigma_T}{\alpha E h} \cdot l^2$$

If the beam is in an elastic state (when $\alpha = 1$),

$$W = -\frac{1}{8} \frac{M l^2}{E I_x} = -\frac{1}{4} \frac{\sigma_T}{E h} \cdot l^2$$

and if in the plastic one ($\alpha = 0$), then $W \rightarrow \infty$, which means that the deflection increases indefinitely.

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