http://doi.org/10.58225/sw.2023.2-203-205

ELASTIC-PLASTIC BENDING OF A RECTANGULAR BEAM

Sazairov Amirkhan Bahsat- PhD. in phy. and math.sc., ass.prof., department of Mechanics,
sazairov_emirxan@mail.ru

Annotations. The article under review examines the stress-strain state of a rectangular beam, referred to a support with hinges at both ends, in flat bending under elastoplastic deformations. As a result, a differential equation for the bent axis of the beam was obtained and solved.
Keywords: elastic-plastic deformation, elastic bending, yield strength

With increasing load, the bending moment in the cross section of the beam reaches the value $M_т = \sigma_т W_x$ ($\sigma_т$ –is the yield strength of the material, W_x –is the axial section modulus), at the most distant points of the section from the neutral axis, the material passes into a plastic state, in accordance with the Prandtl diagram (Fig. 1). Strain begins to increase at constant stress.

With a further increase in the load, plastic deformation gradually occupies an increasing part of the cross section. The diagram of normal stresses divides the section into three zones:in the central elastic core, the stresses change linearly along the ordinate axis, and in the two extreme plastic zones remain constant and equal to $\pm\sigma_т$.

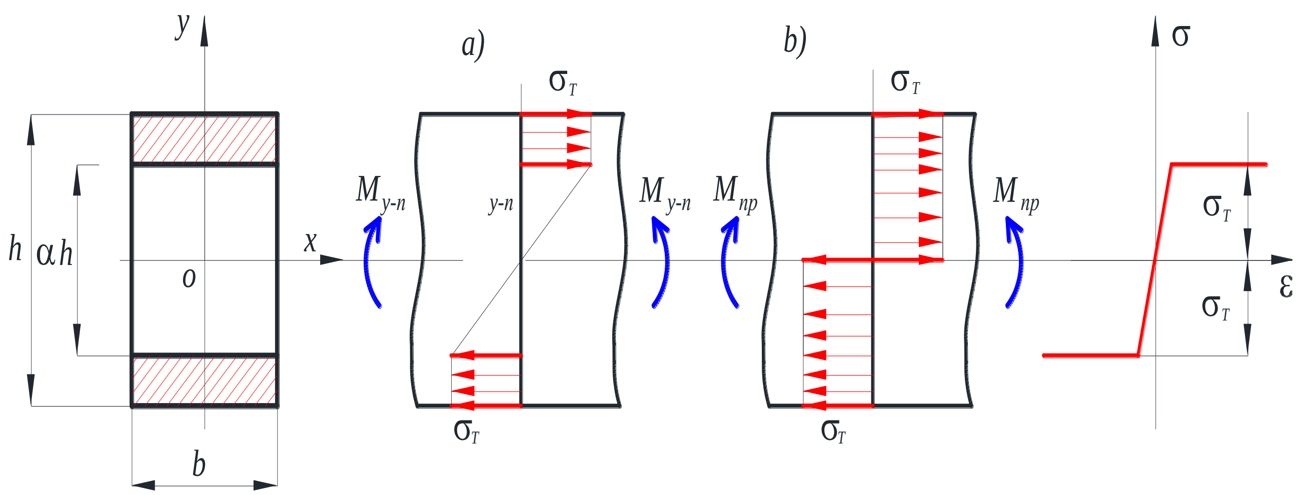

Figure 1. Prandtl diagram [4]

Denote the thickness of the elastic core by αh, depending on the value of the coefficient α, we will distinguish three types of bending:

a) $\alpha = 1$ –elastic bend:

$$M_{упр} = \sigma W_x\,, \quad M_{упр} = M_т = \sigma_т W_x\,, \quad W_x = \frac{bh^2}{6}$$

b) $\alpha = 0$ –plastic bending: $M_{пр} = \sigma_т W_{пл}\,, \quad W_{пл} = \frac{bh^2}{4}$

this state in which the stresses at all points of the section are equal to the yield strength, is called a plastic hinge, unlike a conventional hinge, a is a constant value equal to the limiting moment $M_{пр}$which corresponds to the complete exhaustion of the bearing capacity of a given section.

c) 0<α<1- elastoplastic

Let us determine the bending moment during elastic-plastic bending, for which we will use the stressdiagram from Fig. 1,

$$\text{M}_{\text{у}-\text{п}} = 2\left[b\sigma_{\text{т}}(1-\alpha)\frac{h}{2}\cdot\frac{1}{2}(1+\alpha)\frac{h}{2}+\frac{1}{2}b\sigma_{\text{т}}\alpha\frac{h}{2}\cdot\frac{2}{3}\alpha\frac{h}{2}\right] = \text{M}_{\text{пр}}\left(1-\frac{\alpha^2}{3}\right) \qquad (1)$$

Find the bent axis of the beam and consider an example. Let two identical, oppositely directed moments be applied to the end sections of a hinged beam (Fig. 2.). In the cross section, normal stresses are distributed according to the following law:

$$\sigma = \begin{cases} E\varepsilon & \lfloor y\rfloor \le \dfrac{\alpha h}{2} \\ \sigma_{\text{т}} & \lfloor y\rfloor \ge \dfrac{\alpha h}{2} \end{cases}$$

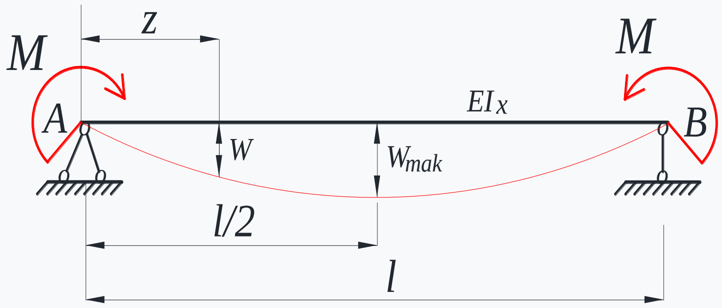

Figure 2. Oppositely directed moments [4]

According to the hypothesis of flat sections, the deformation at the point of the section with the ordinate y,
$\varepsilon = \frac{y}{\rho}$ — here ρ is the radius of curvature of the curved axis of the beam.
Bending moment in section:

$$\text{M} = \int_{A} \sigma y dA = 2\int_{\frac{\alpha h}{2}}^{\frac{h}{2}} \sigma_{\text{т}} y dA + \int_{-\frac{\alpha h}{2}}^{\frac{\alpha h}{2}} E\frac{y}{\rho} y dA == 2\sigma_{\text{т}}\int_{\frac{\alpha h}{2}}^{\frac{h}{2}} y dA + \frac{E}{\rho}\int_{-\frac{\alpha h}{2}}^{\frac{\alpha h}{2}} y^2 dA = 2\sigma_{\text{т}} S_x^{\text{пл}} + \frac{E}{\rho} I_x^{\text{упр}} \qquad (2)$$

Included in this expression $S_x^{\text{пл}}$ —is the static moment of the upper (or lower) plastic zone, and $I_x^{\text{упр}}$ —control is the moment of inertia of the elastic core relative to the neutral axis

$$S_x^{\text{пл}} = \int_{\frac{\alpha h}{2}}^{\frac{h}{2}} bydy = \frac{by^2}{2}\Bigg|_{\frac{\alpha h}{2}}^{\frac{h}{2}} = \frac{bh^2}{8}(1-\alpha^2);\ \ I_x^{\text{упр}} = \frac{b(\alpha h)^3}{12} = \alpha^3 I_x \qquad (3)$$

From equation (2) we find the curvature of the curved axis:

$$\frac{1}{\rho} = \frac{M}{EI_x^{\text{упр}}} = \frac{2S_x^{\text{пл}}\sigma_{\text{т}}}{EI_x^{\text{упр}}} = \frac{\text{M}_{\text{пр}}\left(1-\frac{\alpha^2}{3}-1+\alpha^2\right)}{\alpha^3 EI_x} = \frac{2}{3}\frac{\text{M}_{\text{пр}}}{\alpha EI_x} = \frac{2\sigma_{\text{т}}}{\alpha Eh}$$

This formula can be used to determine the stress in an elastic core:

$$\sigma = \frac{E}{\rho}\cdot y = \frac{2\sigma_{\text{т}}}{\alpha h}\cdot y$$

and in the plastic zone it is constant: $\sigma = \sigma_{\text{т}}$.

Assuming that the displacements are small even under plastic deformations, we write the approximate differential equation of the curved axis:

$$\frac{1}{\rho} = \frac{d^2W}{dz^2} = \frac{2\sigma_т}{\alpha Eh} \tag{4}$$

Separating the variables of the differential equation and integrating twice, we obtain the following equations for the angle of rotation v and deflection W:

$$v = \frac{dW}{dz} = \frac{2\sigma_т}{\alpha Eh} \cdot z + C_1$$

$$W = \frac{\sigma_т}{\alpha Eh} \cdot z^2 + C_1 \cdot z + C_2$$

The constants of integration are found from the boundary conditions: For $z = 0$ and $z = l$ $\rightarrow W = 0$

Substituting the found constants $C_1 = -\frac{\sigma_т}{\alpha Eh} \cdot l$ and $C_2 = 0,$, we finally obtain the equations for the angle of rotation and deflection:

$$v = \frac{\sigma_т}{\alpha Eh}(2z - l)\,; \quad W = \frac{\sigma_т}{\alpha Eh}(z^2 - lz)$$

Maximum deflection (in the middle of the beam, $z = \frac{l}{2}$):

$$W = -\frac{1}{4}\frac{\sigma_т}{\alpha Eh} \cdot l^2$$

If the beam is in an elastic state (when $\alpha = 1$),

$$W = -\frac{1}{8}\frac{Ml^2}{EI_x} = -\frac{1}{4}\frac{\sigma_т}{Eh} \cdot l^2$$

and if in the plastic one $(\alpha = 0)$, then $W \rightarrow \infty,$ which means that the deflection increases indefinitely.

References

1. Aleksandrovich A.I. Ploskaya neodnorodnaya zadacha teorii uprugosti. Vesti. Mosk, matem.,mekh., №1, 1973
2. Kolchin G.B Raschèt elementov konstrukcij iz uprugih neodnorodnyh materialov. Kishinèv 1971
3. Lekhnickij S.G. Zadacha Sen-Venana dlya nepreryvno neodnorodnogo anizotropnogo brusa. Sb. Mekhanika slloshnoj sredy i rodstvennye problemy analiza. Nauka , 576s. 1972
4. Lomakin V. A. Teoriya uprugosti neodnorodnyh tel, Izd-vo Mosk, 368s. 1976
5. Lomakin V. A., Shejnin V.I. O primenimosti metoda malogo parametra dlya ocenki napryazhenij v neodnorodnyh uprugih sredah. Mekhanika tvèrdogo tela, №3, 1972
6. Plevako V.P. K teorii uprugosti neodnorodnyh sred. Prikladnaya matematika i mekhanika, 1971
7. Plotnikov M.M. O napryaazheniyah v odnoj zadache neodnorodno-anizotropnogo cilindra. Izv. vuzov. Mashinostroenie, №8, 1967

Məqaləyə istinad: Sazairov A.B. Düzbucaqlı bir şüanın elastik-plastik əyilməsi. Elmi Əsərlər jurnalı/ Scientific works. AzMİU, s. 203-205, N2, 2023
For citation: Sazairov A.B Elastic-plastic bending of a rectangular beam. Elmi Əsərlər jurnalı/ Scientific works. AzUAC, p. 203-205, N2, 2023

Məqalə INTERNATIONAL CONGRESS ON ADVANCED EARTHQUAKE RESISTANT STRUCTURES (AERS2023) adlı konfrans materialıdır.