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FREE OSCILLATIONS OF A RING-REINFORCED, SHEAR-DAMAGED CYLINDRICAL SHELL IN CONTACT WITH VISCOELASTIC SOIL AND FLUID

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The system we studied consists of a cylindrical shell reinforced with discretely distributed ribs made of viscoelastic, orthotropic material in contact with viscoelastic soil. The incisions are rectangular in shape and located in a section divided by rings, as shown in Figure 1.

To study the oscillations of this system, we will use the full energy of the viscoelastic, orthotropic perforated cylindrical shell reinforced with ribs, as well as the work done in the displacements of the points of the cylindrical shell and the contact conditions added to them. However, the total energy of the system is carried not over the entire S surface, but over the , $S - S_*$ surface, where S_* is the cross-sectional area. It should be noted that the liquid is located inside the cylindrical shell, while the soil is located outside. The liquid is considered ideal, while the soil is considered viscoelastic.

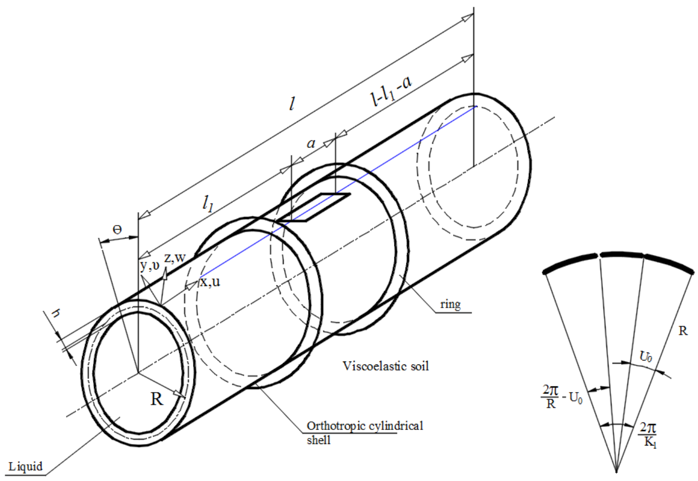

Figure 1. An orthotropic, rectangular cross-sectional viscoelastic cylindrical shell in dynamic contact with fluid and viscoelastic soil, reinforced with rings

The total energy of the system consisting of an anisotropic cylindrical shell reinforced with discretely distributed rings on its surface is given by [1]:

$$\Pi_{dh} = \frac{1}{2}R^2 \iint_{S-S_*} (N_{11}\varepsilon_{11} + N_{22}\varepsilon_{22} + N_{12}\varepsilon_{12} - M_{11}\chi_{11} - M_{22}\chi_{22} -$$

$$M_{12}\chi_{12}\} d\xi d + + \frac{R}{2}\int_0^{2\pi} \left[E_j F_j \left(\frac{\partial \vartheta_j}{R\partial\theta} - \frac{w_j}{R} \right)^2 + \breve{E}_j J_{xj} \left(\frac{\partial^2 w_j}{\partial x^2} + \frac{w_j}{R^2} \right)^2 + \right.$$

$$\left. + \breve{E}_j J_{zj} \left(\frac{\partial^2 u_j}{R^2 \partial\theta^2} - \frac{\varphi_{kpj}}{R} \right)^2 + \breve{G}_j J_{kpj} \left(\frac{\partial \varphi_{kpi}}{R\partial\theta} + \frac{1}{R}\frac{\partial u_j}{\partial y} \right)^2 \right] d\theta + \rho_0 R h \int_0^{\xi_1}\int_0^{2\pi} \left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial \vartheta}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2 \right]$$

$$d\xi\, d\theta +$$

$$+\sum_{j=1}^{k_2}\tilde{\rho}_j F_j R\int_0^{2\pi}\left[\left(\frac{\partial u_j}{\partial t}\right)^2+\left(\frac{\partial \vartheta_j}{\partial t}\right)^2+\left(\frac{\partial w_j}{\partial t}\right)^2+\frac{J_{kpj}}{F_j}\left(\frac{\partial \varphi_{kpj}}{\partial t}\right)^2\right]d\theta-\iint_{S-S_*}(q_z-q_{zz})w\,d\xi\,d\theta \quad (1)$$

The expressions of internal forces and moments are:

$$N_{ij}=\int_{-h/2}^{h/2}(\sigma_{ij}+zw_{ij})dz;\qquad M_{ij}=-\int_{-h/2}^{h/2}(\sigma_{ij}+zw_{ij})zdz \quad (2)$$

$$w_{11}=b_{11}\chi_{11}+b_{12}\chi_{22};\ w_{22}=b_{12}\chi_{11}+b_{22}\chi_{22};\qquad w_{21}=w_{12}=b_{66}\ \chi_{12}.$$

(2) The stresses σ_{ij} and components of the deformation tensor $\varepsilon_{ij}$of the middle surface are included in the expressions as follows:

$$\sigma_{11}=b_{11}\varepsilon_{11}+b_{12}\varepsilon_{22}\qquad \sigma_{12}=b_{66}\varepsilon_{12}\quad \sigma_{22}=b_{12}\varepsilon_{11}+b_{22}\varepsilon_{22} \quad (3)$$

We will take the deformation components $\varepsilon_{ij}\ (i,j=1,2)$ included in (2) and (3) as follows:

$$\varepsilon_{ij}=\tilde{\varepsilon}_{ij}+\int_{-\infty}^{t}\Gamma(t-\tau)\,\tilde{\varepsilon}_{ij}(\tau)d\tau \quad (4)$$

Here

$$\tilde{\varepsilon}_{11}=\frac{\partial u}{\partial x};\ \tilde{\varepsilon}_{22}=\frac{\partial \vartheta}{\partial y}+w;\tilde{\varepsilon}_{12}=\frac{\partial u}{\partial y}+\frac{\partial \vartheta}{\partial x}\qquad \chi_{11}=\frac{\partial^2 w}{\partial x^2};\ \chi_{22}=\frac{\partial^2 w}{\partial y^2};\ \chi_{12}=-2\frac{\partial^2 w}{\partial x\partial y} \quad (5)$$

$\Gamma(t-\tau)$ – is the viscosity core.

In expressions (2) and (3), b_{11}, b_{22}, b_{12} and b_{66} are the main elastic moduli of the cylindrical shell made of orthotropic material, the Young's moduli in the direction of the coordinate lines $E_1,\ E_2$ are expressed as the Young's modulus G and Poisson's coefficients ν_1, ν_2: $b_{11}=\frac{E_1}{1-\nu_1\nu_2};\ b_{22}=\frac{E_2}{1-\nu_1\nu_2};\ b_{12}=\frac{\nu_2 E_1}{1-\nu_1\nu_2}=\frac{\nu_1 E_2}{1-\nu_1\nu_2};\qquad b_{66}=G_{12}=G.$

In expression (1), u,ϑ,w – displacements of the points of the middle surface of the cover, u_i,ϑ_i,w_i – displacements of the shaft points, R,h – the radius and thickness of the cylindrical shell, respectively, $\tilde{E}_j$– Young's modulus of the material of the ring, F_j – j Cross-sectional area of the -th ring, I_{xj},I_{kpj}– moments of inertia of the cross-section of the j-th ring $\tilde{G}_j$ – Jung's modulus of the j-th ring in sliding, k_2– number of rings, *t* – time, ρ_0– of the cylindrical cover, $\tilde{\rho}_j$- j - are the densities of the materials of the th ring.

It is assumed that the following rigid contact conditions between the cylindrical cover and the rods are satisfied [2]:

$$u_i(x)=u(x,y_i),\vartheta_i(x)=\vartheta(x,y_i)\ ,w_i(x)=w(x,y_i) \quad (6)$$

It is assumed that the anisotropic cylindrical cover is finite and its ends are connected by joints. That is, the following conditions are satisfied at the edges

$$\xi=\ 0\ \text{və}\ \xi=\xi_1:\vartheta=w=0;\ T_1=\ 0;\ M_1=\ 0 \quad (7)$$

The effect of the soil and liquid on the cylindrical cover q_z,q_{zz} is calculated as follow

$$A_{0d}=-\iint_{S-S_*}(\,q_z-q_{zz}\,)\ wd\ \xi\ d\ \theta \quad (8)$$

Here q_z is the normal force acting on the cylindrical shell by the viscoelastic soil and is calculated as follows:

$$q_z=kw+\int_{-\infty}^{t}\Gamma_*(t-\tau)\,w(\tau)d\tau$$

(9)

Here $\Gamma_*$is the (t-τ)-viscosity kernel.

Since the force does not act on the section, the work done by them in the displacement of the points of the cylindrical shell will be zero.

As a result, the solution of the considered problem is the (1) total energy (6) contact and (7) boundaries of the structure consisting of a cylindrical cover with a rectangular cross-section, which is in dynamic contact with the visco-elastic soil and liquid, and the anisotropy and visco-elastic properties of its material are taken into account. is brought to joint integration within the conditions.

We will look for the components of the shell displacement vector as follows: $u = u_0 \sin\chi\xi \cos n\theta \sin\omega t$

$$\vartheta = \vartheta_0 \cos\chi\xi \sin n\theta \sin\omega t \tag{10}$$

$w = w_0 \cos\chi\xi \cos n\theta \sin\omega t$

Here $\xi = \frac{x}{l}$, u_0, ϑ_0, w_0- are unknown constants.

Using the property of the integral $\iint_{S-S_*} = \iint_S - \iint_{S_*}$ and the solutions of (12), the pressure force applied to the cylindrical shell from the soil and liquid sides is seen in the displacements of the points of the shell we can calculate the work. Since S* is a rectangular area, we can write $(l_1 \le x \le l_1 + a, \varphi_0 R \le b \le \frac{2\pi}{k_1} - \varphi_0 R)$,:

$$A_{0d} = \frac{\pi\xi_1}{2}\phi_{an}\rho_m\left(\omega^2 - \frac{U^2\chi^2}{R^2}\right)w_0^2 \sin^2\omega t + \left(k\sin^2\omega t + \int_{-\infty}^{t}\Gamma_*(t-\tau)\sin^2\omega\tau d\tau\right)\times \tag{11}$$
$$\times\left\{\frac{1}{2}\pi l - \frac{1}{2}a\left[\frac{\pi R}{k_1} - \varphi_0 R + \frac{1}{n}\sin 2n\left(\frac{\pi R}{k_1} - \varphi_0 R\right)\cos\frac{4n\pi R}{k_1}\right] + \frac{1}{2\chi}\sin 2\chi a \cos 2\chi(2l_1 + a)\right\}w_0^2$$

Substituting expressions (10) into (1) and considering (11), we obtain for the total energy of an orthotropic viscoelastic cylindrical shell with a rectangular cross-section, reinforced with rings, in dynamic contact with viscoelastic soil and a liquid:

$$\Pi_{hd} = \tilde{d}_{11}u_0^2 + \tilde{d}_{22}\vartheta_0^2 + \tilde{d}_{33}w_0^2 + \tilde{d}_{12}u_0\vartheta_0 + \tilde{d}_{13}u_0 w_0 + \tilde{d}_{23}\vartheta_0 w_0 \tag{12}$$

$$\tilde{d}_{11} = \left[\frac{h}{2R}(b_{11}\chi^2 + b_{66}n^2)\left(\frac{\pi}{2\omega} + F(\omega)\right) + \rho_0\frac{h}{R}\omega^2\right]\times$$
$$\times\left\{\frac{1}{2}\pi l - \left[\frac{\pi R}{k_1} - \varphi_0 R + \frac{1}{n}\sin 2n\left(\frac{\pi R}{k_1} - \varphi_0 R\right)\cos\frac{4n\pi R}{k_1}\right]\times\right.$$
$$\left.\times\frac{1}{2}a + \frac{1}{2\chi}\sin 2\chi a \cos 2\chi(2l_1 + a)\right\} + \frac{\pi}{2\omega R}\sum_{j=1}^{k_2}\frac{\breve{G}_j n^2}{R^2\breve{E}_j}J\cos^2\frac{k\xi_j}{L}{}_{kp.j};$$

$$\tilde{d}_{22} = \left[\frac{h}{2R}(b_{22}n^2 + b_{66}\chi^2)\left(\frac{\pi}{2\omega} + F(\omega)\right) + \rho_0\frac{\pi}{2}\frac{h}{R}\omega\right]\times$$
$$\times\left\{\frac{1}{2}\pi l - \left[\frac{\pi R}{k_1} - \varphi_0 R + \frac{1}{n}\sin 2n\left(\frac{\pi R}{k_1} - \varphi_0 R\right)\cos\frac{4n\pi R}{k_1}\right]\times\frac{1}{2}a + \right.$$
$$\left.+\frac{1}{2\chi}\sin 2\chi a\cos 2\chi(2l_1 + a)\right\} + \frac{\pi}{2R}\sum_{j=1}^{k_2}\breve{E}_j F_j \sin^2\frac{k\xi_j}{L} - \omega^2\pi\sum_{j=1}^{k_2}\tilde{\rho}_j F_j\cos^2\chi\xi_j$$

$$d_{33} = \left\{\frac{h}{2R}\left[-\frac{h}{4}\chi^2 b_{12} - \frac{h}{4}b_{22}n^2 + \frac{h^2}{12}(n^2\chi b_{12} + n^4 b_{22}) + \frac{h^2}{3}\chi^2 n^2 b_{66} + \right.\right.$$
$$\left.+b_{22}\right]\left(\frac{\pi}{2\omega} + F(\omega)\right) + \rho_0\frac{h}{2R}\omega\Big\}\left\{\frac{1}{2}\pi l - \left[\frac{\pi R}{k_1} - \varphi_0 R + \frac{1}{n}\sin 2n\left(\frac{\pi R}{k_1} - \right.\right.\right.$$
$$\left.\left.\left.-\varphi_0 R\right)\cos\frac{4n\pi R}{k_1}\right]\times\frac{1}{2}a + \frac{1}{2\chi}\sin 2\chi a\cos 2\chi(2l_1 + a)\right\} +$$
$$+\left[\frac{\pi}{2R}\sum_{j=1}^{k_2}\left(\breve{E}_j F_j\frac{h_j^2 n^4}{R^2} + \frac{J_{xj}}{R^2}(1-n^2)^2\right)\sin^2\frac{k\xi_j}{L} + \frac{\breve{G}_j}{R^2\breve{E}_j}J_{kp.j}n^2k^2\cos^2\frac{k\xi_j}{L} - \right.$$
$$\left.-\omega^2\pi\sum_{j=1}^{k_2}\tilde{\rho}_j F_j\cos^2\chi\xi_j\right]\frac{\pi}{2\omega}\Bigg\};$$

$$\tilde{d}_{12} = 2n\chi \frac{\pi Lh}{4R}(b_{12}+b_{66})\left(\frac{\pi}{2\omega}+F(\omega)\right)\left\{\frac{1}{2}\pi l - \left[\frac{\pi R}{k_1} - \varphi_0 R + \frac{1}{n}\sin 2n\left(\frac{\pi R}{k_1} - \right.\right.\right.$$
$$\left.\left.\left. - \varphi_0 R\right)\cos\frac{4n\pi R}{k_1}\right] \times \frac{1}{2}a + \frac{1}{2\chi}\sin 2\chi\, a\cos 2\chi(2l_1+a)\right\}$$

$$\tilde{d}_{13} = \left(-2\chi b_{12} + \frac{h}{4}\chi^3 b_{11} + \frac{h}{4}n^2\chi b_{12} - \frac{h}{2}b_{66}\chi n^2\right)\left(\frac{\pi}{2\omega}+F(\omega)\right)\times$$
$$\times\left\{\frac{1}{2}\pi l - \left[\frac{\pi R}{k_1} - \varphi_0 R + \frac{1}{n}\sin 2n\left(\frac{\pi R}{k_1} - \varphi_0 R\right)\cos\frac{4n\pi R}{k_1}\right]\times\right.$$
$$\left.\times\frac{1}{2}a + \frac{1}{2\chi}\sin 2\chi a\cos 2\chi(2l_1+a)\right\} + \frac{\pi^2}{2\omega R}\sum_{j=1}^{k_2} F_j \frac{n^2 k\breve{G}_j}{R^2} J\cos^2\frac{k\xi_j}{L_{kp.j}}$$

$$\tilde{d}_{23} = \left(2n b_{22} - \frac{h}{4}n\chi^2 b_{12} + \frac{h}{4}b_{22}n^3 + \frac{h}{2}b_{66}n\chi^2\right)\left(\frac{\pi}{2\omega}+F(\omega)\right)\times$$
$$\times\left\{\frac{1}{2}\pi l - \left[\frac{\pi R}{k_1} - \varphi_0 R + \frac{1}{n}\sin 2n\left(\frac{\pi R}{k_1} - \varphi_0 R\right)\cos\frac{4n\pi R}{k_1}\right]\times\right.$$
$$\left.\times\frac{1}{2}a + \frac{1}{2\chi}\sin 2\chi a\cos 2\chi(2l_1+a)\right\} - \frac{\pi^2}{2\omega R}\sum_{j=1}^{k_2}\frac{h_j n^2}{R}\breve{E}_j F_j \sin^2\frac{k\xi_j}{L}$$

If we vary the equation (12) according to the free quantities u_0, ϑ_0, w_0 and take the coefficients of the free variations to be zero, we get the following system of homogeneous algebraic equations. Since the obtained system is a system of linear homogeneous algebraic equations, it is a necessary and sufficient condition that the principal determinant is equal to zero for the existence of its non-trivial solution. As a result, we get the following frequency equation:

$$8\tilde{d}_{11}\tilde{d}_{22}\tilde{d}_{33} + \tilde{d}_{22}\tilde{d}_{23}\tilde{d}_{13} + \tilde{d}_{12}\tilde{d}_{23}\tilde{d}_{12} -- 2\tilde{d}_{22}\tilde{d}_{13}^2 - 2\tilde{d}_{11}\tilde{d}_{23}^2 - 2\tilde{d}_{33}\tilde{d}_{12}^2 = 0 \qquad (13)$$

Equation (13) is a transcendental equation with respect to the unknown ω. Its roots are found by the numerical method. In the calculation process, the quantities determining the cover, ribs, cross-section dimensions and soil were taken as follows[3,4]: $\rho_0 = \rho_j = 0{,}26 \cdot 10^4$ Nsan2/m4, $\beta =$ 0,05; $E_j = 6{,}67 \cdot 10^9 \frac{N^2}{m}$; $\nu = 0{,}3$;

$k = 5{,}3 \cdot 10^7 N/m^2; \varphi_0 = \frac{\pi}{72}$; a = 50*mm*

The results of the calculation are given in the form of dependence of ω_1 on the wave number *n* in figure 2, viscosity parameter of ω_1 in figure 3, and the number of transverse ribs κ_2 in figure 4 b_{11} =18,3 *QPa*; b_{22} =25,2 *QPa*; b_{66}=3,5 *QPa*; b_{12}=2,77 *QPa*; $\psi = \psi_1$ 0,05 is taken.

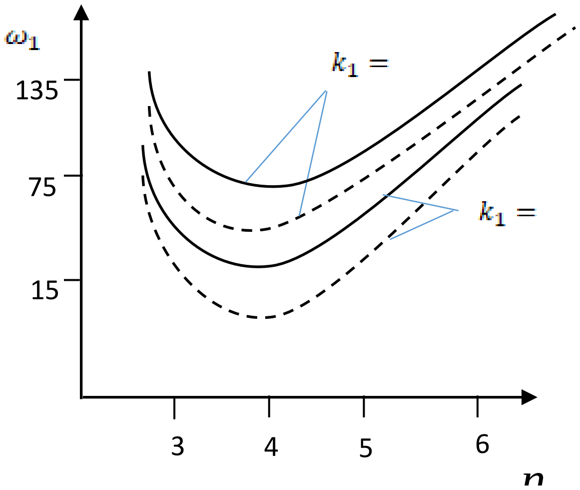

Figure 2. Dependence of frequencies on wave number *n*

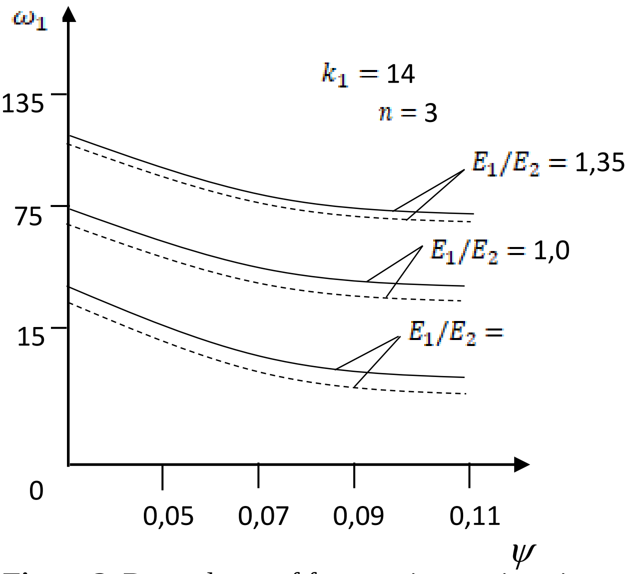

Figure 3. Dependence of frequencies on viscosity parameter ψ

Figure 2 shows as the wave number n increases in the circular direction, the true frequencies of the structure decrease and take a minimum value and increase again. In this way, $\psi = \psi_* = 0$ corresponds to solid lines, and $\psi = \psi_* =$ 0,05 to broken lines.

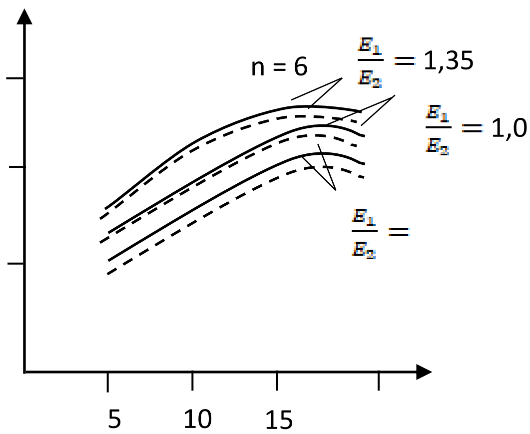

Figure 4. Dependence of frequencies on the number of rings k_2

As shown in Figure 3, an increase in the viscosity parameter leads to a decrease in the frequencies corresponding to the characteristic oscillations of the structure. Figure 4 demonstrates that an increase in the number of rings initially causes an increase in the frequencies corresponding to specific oscillations of the structure, but after a certain increase in the number of transverse ribs, the frequencies begin to decrease. This occurs because the mass of the transverse ribs increases, which strengthens the effect of inertia on the oscillation process. The broken lines in Figures 3 and 4 represent the oscillations of the cylindrical shell attenuated by the hole, while the solid lines represent the oscillations of the intact cylindrical shell. It is evident from the figures that the oscillation frequencies of the hole-attenuated cylindrical shell are lower than those of the intact cylindrical shell. It is worth noting that the holes can be in other shapes, such as circles, and the solution method for these cases is similar to that for rectangular holes, with the only difference being that the domain S_i is circular. Additionally, multiple holes can be opened on the surface of the cylindrical shell without introducing any fundamental difficulty in solving the problem. The only change is that the number of integration areas S_i increases.

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